Using a Multi-objective Genetic Algorithm for Curve Approximation

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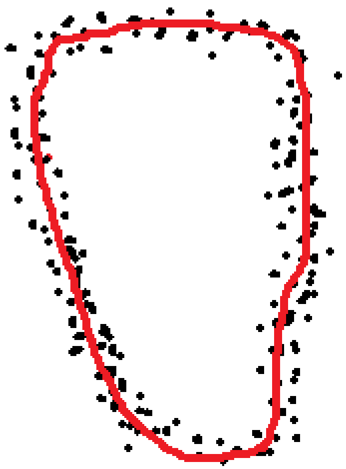
Contents

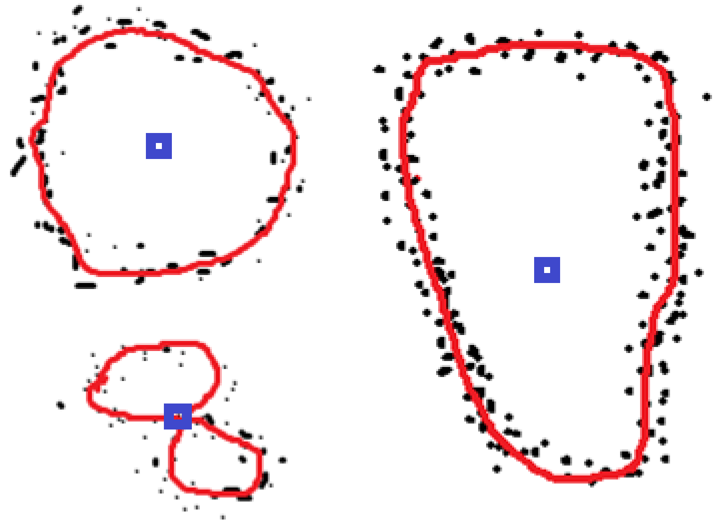
Curve Fitting

- Approximate an (un-)ordered set of points / voxels with a curve
- Many algorithms require vector data such as lines, curves or surfaces
- B-splines are of special interest for applications in computer-aided geometric design
  - Their piecewise polynomial structure makes them flexible and quick to compute
  - Their structure can be easily extended to surface and hyperplanes



What do we need it for?

- This work was done to find cluster descriptors for density-based clusters.
- Those can be efficiently mined, but most prototype methods provide unsuitable approximations.
- E.g. consider the mean vector as it is commonly used for clusters found by k-means



Mean vector

True approximation of the cluster form

Previous strategies – Related Work I

- Regression methods
  - Requires the order of points to be known
- Genetic algorithms have been successfully applied for the determination of optimal number and location of knots
  - Requires the order of points to be known
- Iterated Thinning / Skeleton construction / Graph construction
  - Restricted to low dimensional data
- … or cannot fit data of certain geometric properties

Previous strategies – Related Work II

- Local Tangential Flow
  - Deals with self intersections, sharp corners and high dimensions
  - But fails in point clouds of varying density

- PCA used for ordering the points on a polynomial chain
  - Fast and deals will high dimensions and self-intersections
  - Struggles with point clouds of varying density

- Nevertheless, density based clusters need efficient descriptors as well

B-Spline Curve Fitting

- Task: find a B-spline curve that approximates the shape best

$$C(t) = \sum_{i=0}^{n} N_{i,k}(t) P_i$$

$P = (P_0, P_1, \dots, P_n)$ is the set of control points

$N_{i,k}(t) = (N_{0,k}(t), N_{1,k}(t), \dots, N_{n,k}(t))$ is the set of B-spline basis functions defined over a degree *k* - 1 and a non-decreasing sequence of real numbers

$T = (u_0, u_1, \dots, u_{n+k})$ is the knot vector.

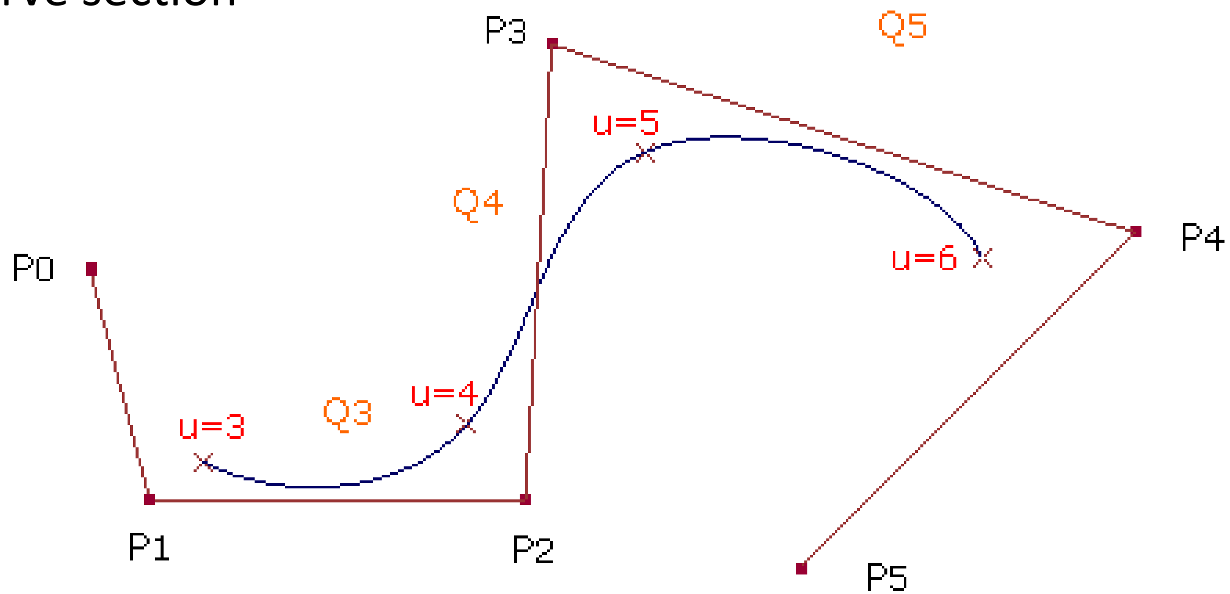
B-Spline Curve Fitting

- A curve is said to be clamped uniform, if the first and last knot in the knot vector each has multiplicity k, and the remaining knots are evenly spaced.
  - Such a curve starts at the first control point and ends at the last control point.

- Due to the definition of the basis functions, a control point P_i only influences the curve in the interval $[u_i; u_{i+k})$ (local control property).

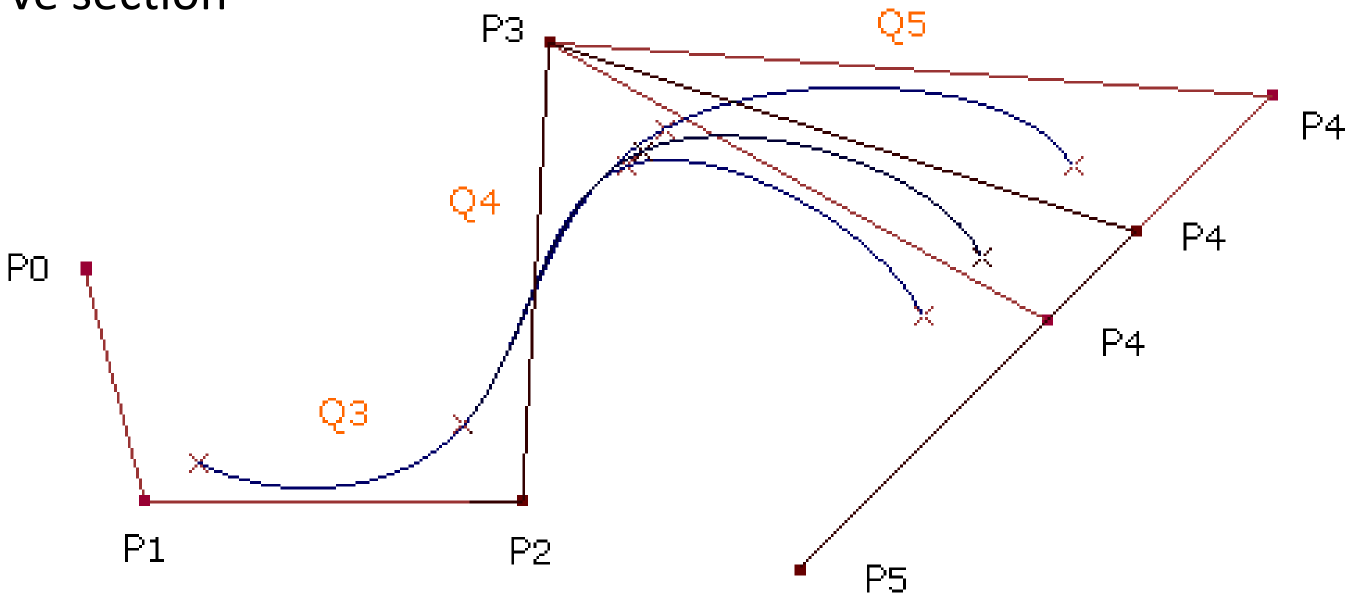
B-splines by example I

- P = control point
- u = knot vector
- Q = curve section



B-splines by example II

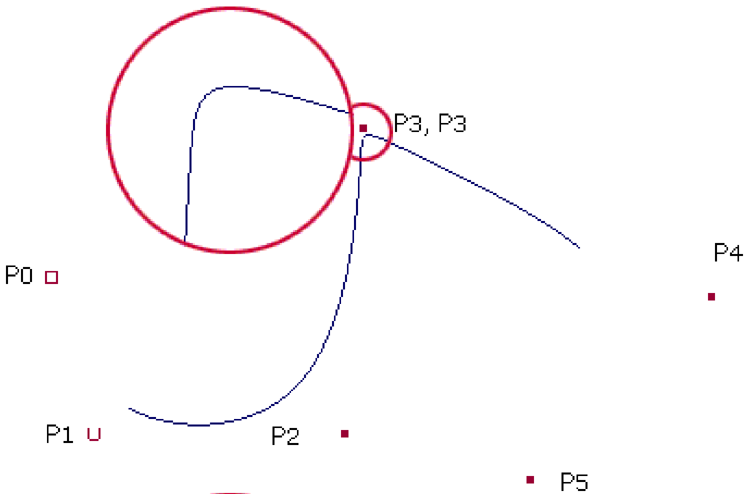
- P = control point
- u = knot vector
- Q = curve section

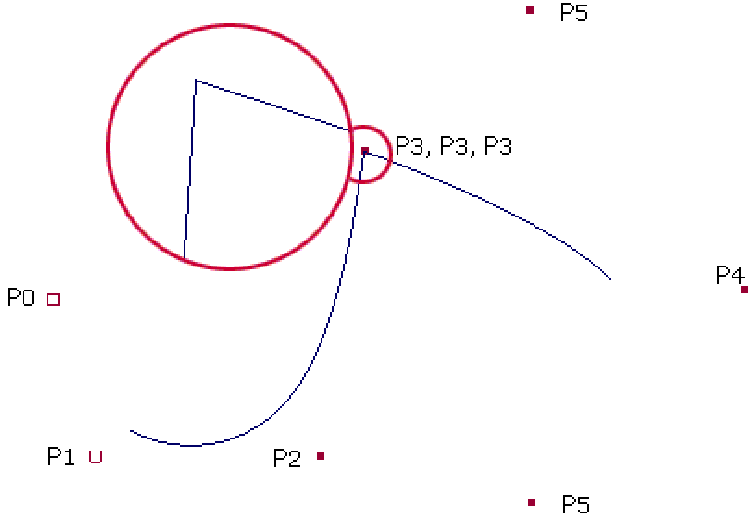


B-splines by example III

- P = control point
- u = knot vector
- Q = curve section

- Playing multiple control points at the same position, sharpens the curve at this section





Multi-objective Optimization with NSGA-II

- During the multi-objective optimization we want to find
  - the best fitting set of control points per b-spline
  - The best set of b-splines to approximate the dataset
- For smoothness: the curves developed in the algorithm are fixed to be cubic, i.e. of degree 3
- For control: changing a control point will only influence the curve in an interval of four knots
- The knot vector

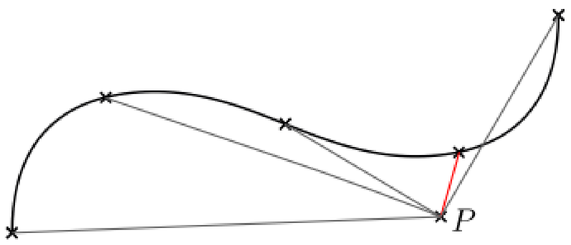
Parameter	Value
Number of generations	1000
Population size	100
Crossover probability	0.10
Probability of mutating control point number	0.01
Probability of mutating control point location	0.25

Objectives

- Objectives used in this study:
  - Number of control points
  - Distance minimization
  - (Length of the curve)

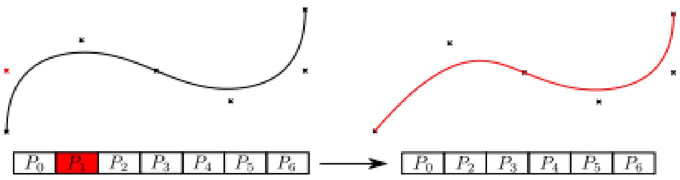
- Other possible objectives:
  - Near-parallel segments
  - Segments outside the points cloud

- Distance calculation:
  - Equidistant sampling of a number of points on the curve
  - Shortest distance to the target point is considered to be the distance of the curve and the point

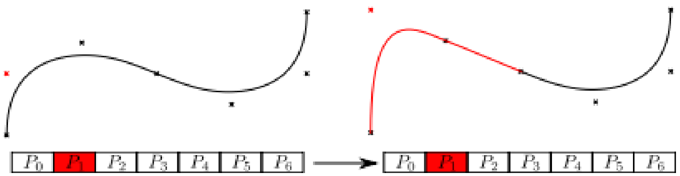


Mutation and Crossover for B-splines

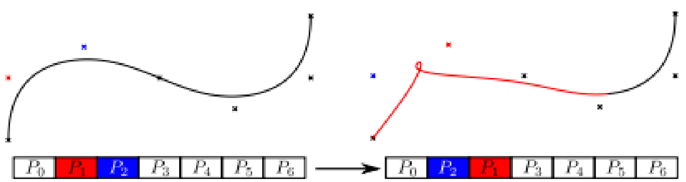
- Mutations:
  - Removing/adding a control point



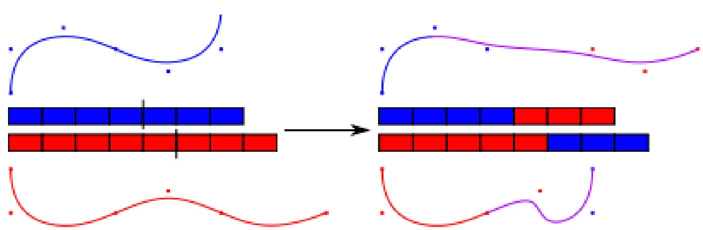
  - Changing a control point via random gaussian position change



  - Swapping the position of two control points

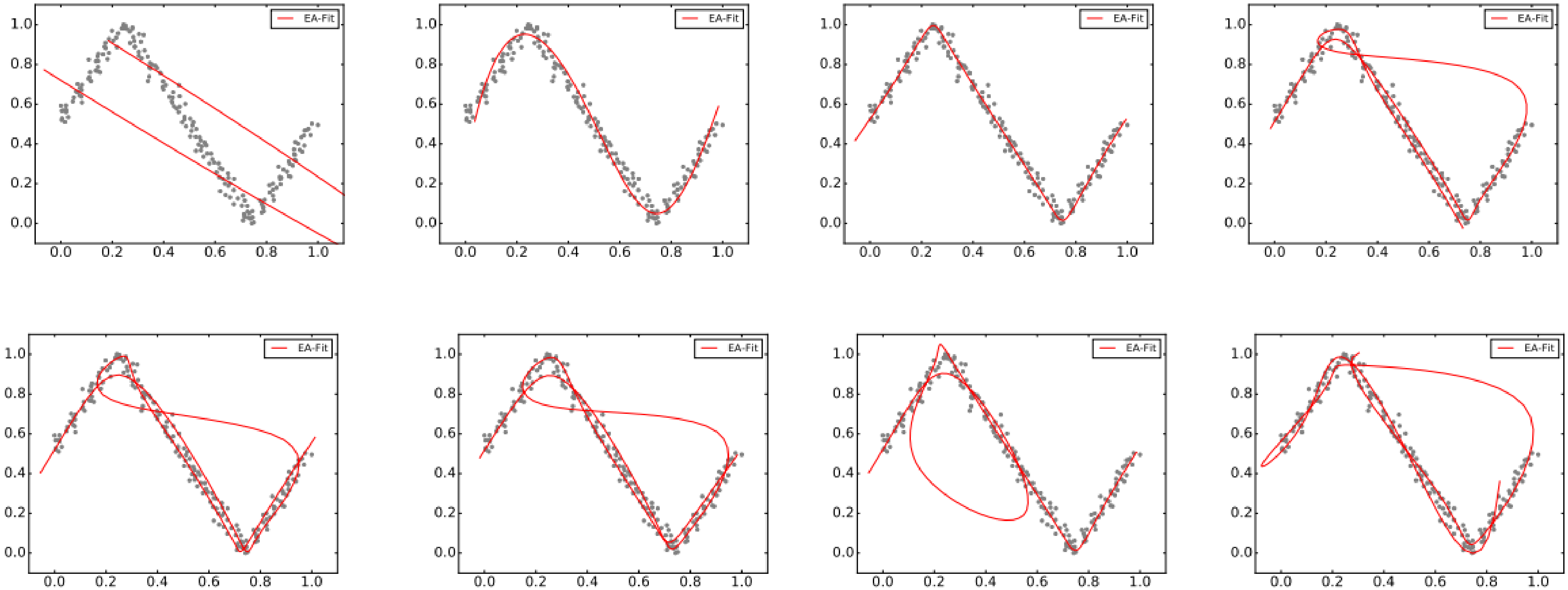


- Crossover:
  - One-point-Crossover



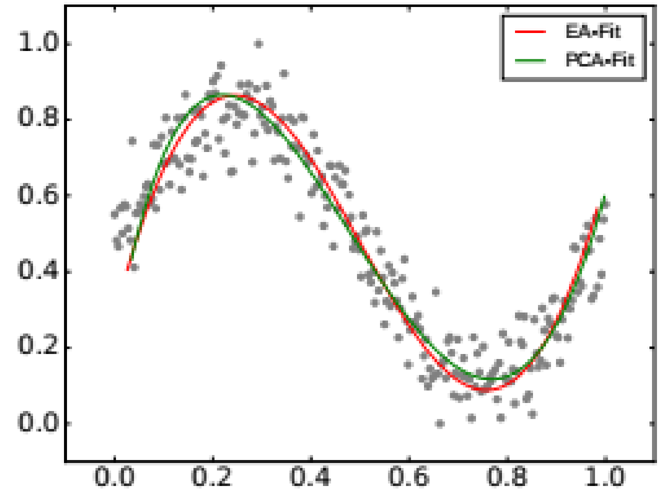
Results of one Pareto Front

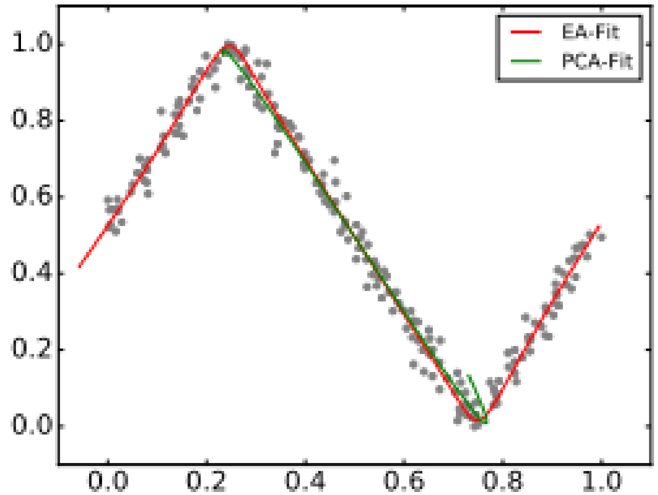
- Models are differing in complexity and error-rate

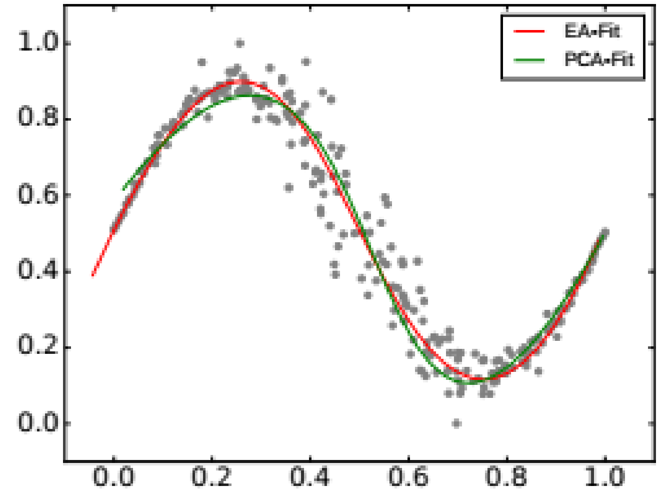


Comparing the Results - I

- Stable results for non-intersecting point-clouds
- Even with varying density and sharp edges, no problems arise





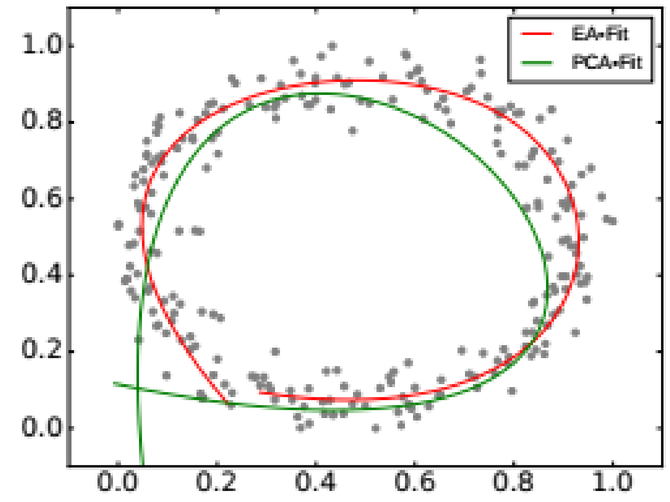


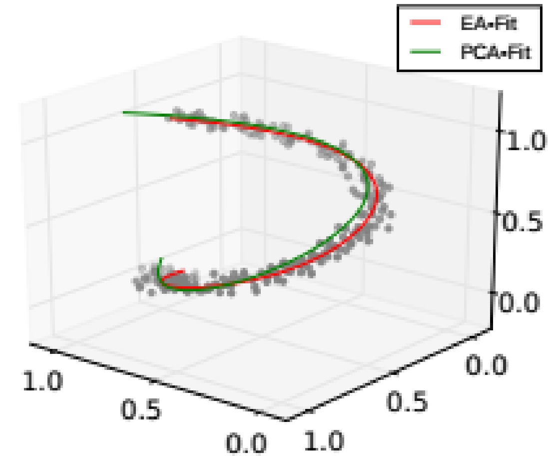
Comparing the Results - II

- Closed forms can be approximated very well

- Few dimensions can be handled without problems

- Multiple dimensions can affect the objectives due to the curse of dimensionality





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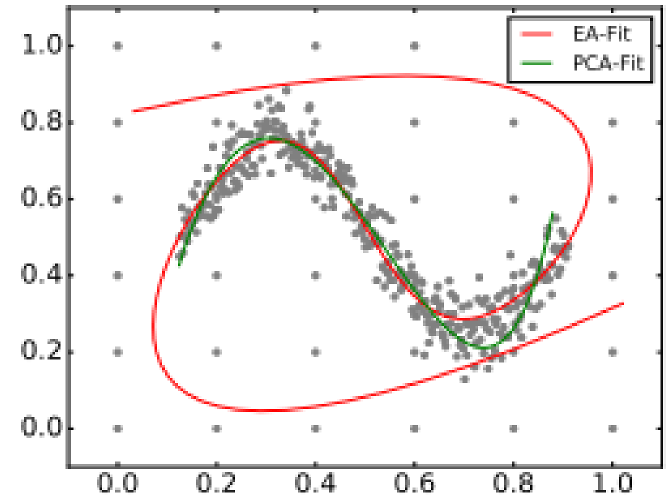
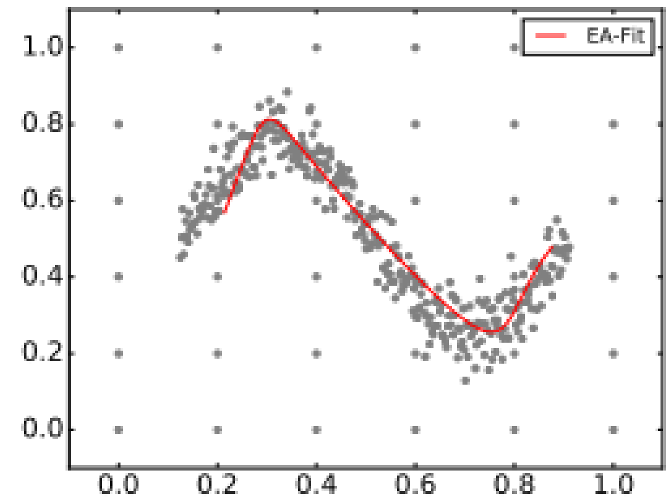
Comparing the Results - III

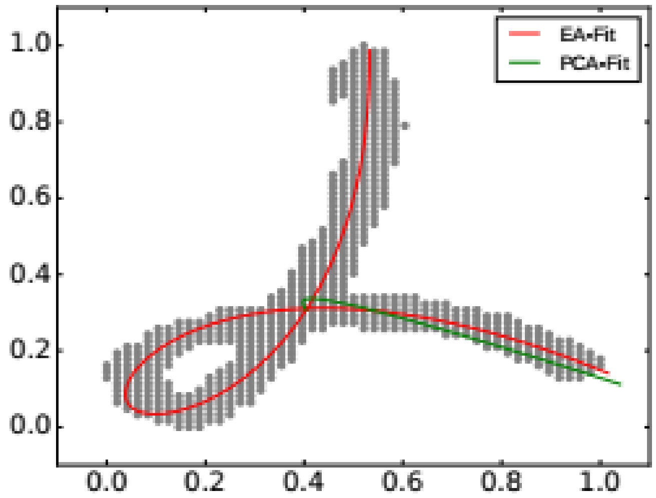
- Background noise can increase the complexity of the resulting model
- Dense areas will be fitted very well, whereas sparse areas are only fitted approximately





Comparing the Results - III

- Intersection can be succesfully modeled
- Nevertheless it becomes exceedingly hard to choose the best fitting model in this case.



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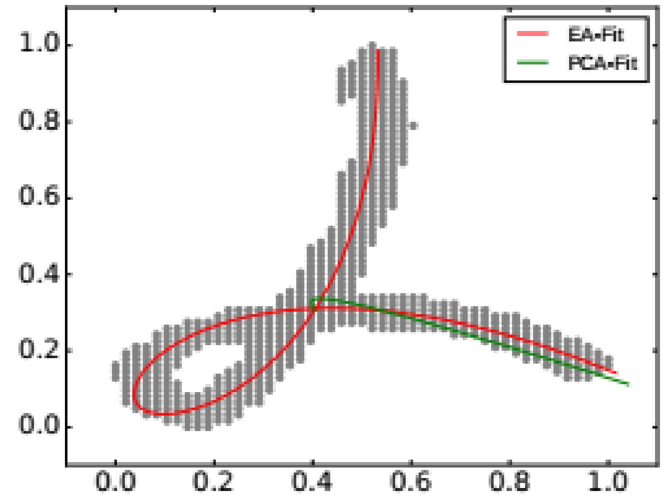

Comparing the Results - IV

- Sum of squared errors per algorithm on the presented datasets

Data Set	EA-Fit	PCA-Fit
Open Point Cloud	**0.4314**	0.4806
Closed Point Cloud	**0.4744**	1.3197
Varying Density	**0.2500**	0.3617
Sharp Corners	**0.0577**	6.0421
Self-Intersections	**0.6195**	53.5088
Three-Dimensional Data	**0.6819**	1.1549
Background Noise	**0.8018**	2.5212

Conclusions

- A survey of existing research revealed that many publications deal with the problem of approximation, but often struggle with point clouds of high dimensions or specific geometric characteristics.

- Two properties of fitting curves were defined as objective functions for the evolutionary
algorithm: the distance to the point cloud as well as the number of control points to preserve simplicity

- based on the multi-objective genetic algorithm NSGA-II

- Critical characteristics, such as non-uniform noise, sharp corners and self-intersections as well as higher dimensional data was fitted properly

Limitations and Future Work

- optimization process is constrained to the control point vector
  - Include the knot vector as well
- The best individual is currently chosen by hand or by the average rating
  - study the influence of each criterion and find better weights thereby
  - Find a method to receive the most suitable individual in the pareto-front
- For a higher number of objectives, one should consider using a Many-Objective Genetic Algorithm, such as NSGA-III
- b-splines cannot represent simple curves like circles and ellipses
  - Adapt the system to non-uniform rational b-splines (NURBS)

Thank you for your attention!

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